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NOTES

ON THE

THEORY OF THE STEAM ENGINE:

BEING PART OF

A Course of Instruction in the Subject

GIVEN IN

THE ROYAL SCHOOL OF NAVAL ARCHITECTURE
AND MARINE ENGINEERING.

BY

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MDCCCLXXI.

INTRODUCTION.

The mechanical theory of heat has now for some years attained a well-established position, while its consequences are so important in the theory of heat engines, that (if true) no theory not based on it could be accepted, even provisionally, as the subject of a course of instruction in the theory of the steam engine. No doubt many numerical results are but slightly altered by the adoption of the true theory of heat, but the whole conception of the mode of operation of a heat engine is altered, and more especially we perceive with clearness, by aid of this theory, the conditions of maximum efficiency, and consequently the means whereby maximum efficiency may be attained. And for this is necessary, not merely a knowledge of the principle of the convertibility of heat and work in which the mechanical theory of heat is too often supposed alone to consist, but also a knowledge of certain other parts of that theory, not less important, though unfortunately much more difficult to understand and apply. Accordingly, the sole English work on the steam engine which is founded on the true theory of heat, namely, Professor Rankine's well-known treatise on the "Steam Engine and other Prime Movers," has always been employed as the text-book of the theory of the steam engine, in the Royal School of Naval Architecture and Marine Engineering.

As may well be supposed, however, a systematic course of oral instruction has been found necessary, on account of the very difficult form in which many parts of the subject are presented in the work in question. But the inconvenience to students and instructor of too great a dependence on oral instruction is so serious, that it was determined to draw up notes in which that part of the instruction, in which the difficulty had been specially felt, might be presented in a definite form.

We have been led to publish these notes, with some additions, from the idea that they might prove useful to persons who, although well acquainted with the steam

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engine and with the principle of the convertibility of heat and work, may have been deterred from a further consideration of the subject by the difficult manner in which the mechanical theory of heat is treated, not only in Rankine's work, but in almost every other work, English or foreign, with which we are acquainted. We have dwelt, therefore, exclusively on points *peculiar* to the modern theory of the steam engine, omitting all others, though of equal importance; for to consider every point would be to write a complete treatise, a work of no small magnitude, which we have no leisure to undertake. It will be found, however, that no such point of much importance is left unnoticed, unless it be fully explained in Rankine's work. Sometimes a point is noticed for the sake of readers who may have at hand no other treatise on the steam engine but Rankine's. We are far from pretending to have removed all difficulties,—the nature of the subject forbids that,—but we hope that we may have succeeded so far in mitigating them as to render them not insuperable by those who are accustomed to reasoning on mechanical questions. It is to be regretted that the expansion curve of steam, when expanding without gain or loss of heat, cannot be determined in a simpler way than that adopted in Art. 26. If a graphical method should ever be devised of solving this problem, the theory of the steam engine would be complete without the aid of a differential equation, and the remaining difficulties would be solely due to the novelty and generality of the ideas involved.

In preparing these notes, considerable use has been made of a work,* which appears to be little known in England, by Dr. Zeuner, till recently a Professor in the Polytechnic School at Zurich, a writer already favourably known by the translation, lately executed, of his work on "Valve Gears." Zeuner's work is extremely valuable to the student of the theory of the steam engine, from the fulness and clearness with which certain parts of the mechanical theory of heat are discussed, and from the unsparing labour which has been bestowed on the wearisome numerical calculations incidental to the subject. Our obligations to this writer will be repeatedly acknowledged in the course of our work.

In dealing with the "second principle" we have followed a course resembling that adopted by M. Verdet, in his well known "Exposé de la théorie mécanique de la chaleur,"† considering at some length the theory of a perfect gas engine before attempting an explanation of the second principle itself. It appears to us that this method admits of being made as logically accurate as any other, while it certainly possesses great advantages in point of simplicity.

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* "Grundzüge der mechanischen Wärmetheorie," von Dr. Gustav Zeuner. Leipzig, 1866.

† "Leçons de chimie et de physique." Paris, 1862.

In Art. 11 we have calculated the amount of heat requisite to keep steam dry as it expands; and finding it always positive, we have inferred that condensation must take place when steam expands without gain or loss of heat. We have not been able to find anywhere this simple method of demonstrating the well-known fact, all demonstrations that we have seen depending on a knowledge of the adiabatic relation.

We shall be glad if our work be accepted as a contribution towards a really elementary statement of the mechanical theory of heat; and we hope it may be of service to students of this important subject, more especially in its application to the theory of the steam engine. After what has been said, it is hardly necessary to state that these notes form but a part of the instruction in this subject actually given in the Royal School of Naval Architecture and Marine Engineering.

Royal School of Naval Architecture,
November, 1871.

ON THE THEORY OF
THE STEAM ENGINE.

UNITS OF MEASUREMENT. PHYSICAL PROPERTIES OF STEAM.

1. From the principle that heat and work are mutually convertible, it follows that quantities of heat may be expressed in foot pounds, and conversely, that quantities of work may be expressed in thermal units. In thermo-mechanical questions it is necessary to have a common unit of measurement for heat and work, and in Rankine's work the foot pound has been adopted, but the thermal unit might likewise have been adopted, in which case quantities of work would be expressed in thermal units by dividing by 772: thus, 1-horse power is represented by $\frac{33000}{772}$ or 42·75 thermal units per minute. This mode of measurement possesses the advantage of leading to smaller numbers than the other; but, nevertheless, it will be seen hereafter that the foot pound is the most convenient unit.

In measuring temperature we employ Fahrenheit's scale as the one in common use, although the centigrade scale is far more convenient, and it is to be hoped may ultimately be adopted.

2. The density of a vapour or gas is best measured by the volume which one pound of it occupies, for which the convenient term "specific volume" is used by Zeuner. In the case of saturated steam the specific volume is given by the approximate formula $pv^{\frac{17}{16}} =$ constant, up to a pressure of 120 lbs. per square inch. (Rankine, p. 403.) The elaborate calculations of Zeuner have fully confirmed this formula as representing with accuracy the density of steam as calculated from the relation existing between its latent heat of evaporation, temperature, and specific volume.

According to him, the agreement (from .5 to 14 atmospheres) is almost exact, if the index 1·0646 be used in place of Rankine's index $\frac{17}{16} = 1{\cdot}0625$. (Mechanische Wärme Theorie, p. 294.) The value of the constant is about 475, if the pressures be in pounds per square inch, and the volumes in cubic feet. If we employ Zeuner's index, the formula, when adapted to logarithmic computation, becomes

$$\log v = 2.516 - .939 \log p$$

On account of the difficulty of obtaining steam in a perfectly dry and saturated condition, the density of steam can hardly be said to have been determined experimentally in a thoroughly satisfactory manner. The best experiments are those made by Fairbairn and Tate, who have given the following formula to represent them :—

$$v = {\cdot}41 + \frac{389}{p - {\cdot}35}$$

where v is the specific volume in cubic feet, and p is the pressure in lbs. per square inch. This formula is very convenient in calculation, but can only be used for pressures between about 20 and 60 lbs. per square inch (absolute): below 20 its results are much too large, above 60 they are much too small. (Comp. Art. 24.)

The density of steam is likewise frequently measured by its "relative volume," that is, by the ratio which its volume bears to the volume of the water from which it was produced. To obtain the relative volume we have simply to multiply the specific volume by 62.5.

The complex relation between the temperature and pressure of saturated steam can only be expressed for a wide range of pressure by formulæ, such as that given by Rankine (p. 237), which require very tedious calculations. For pressures between 6 and 60 lbs. per square inch we may use the formula

$$p = \left(\frac{t + 40}{147}\right)^5$$

where p is the pressure in pounds per square inch, and t is the temperature Fahrenheit. From 60 to 120 lbs. per square inch this formula may likewise be used, but unity should then be added to the result.

Rankine has given a table of the pressure corresponding to a given temperature, but the table unfortunately only extends to every 9° F. For the convenience of students a table is subjoined of the temperature corresponding to a given pressure for every pound on the square inch, from 1 to 120. Fuller tables in English measures, which appear accurate so far as the writer has been able to examine them, will be found in "Porter, on the Richards Indicator." The diagram recently published by Rankine is likewise convenient for obtaining the pressure and specific volume corresponding to a given temperature.

TABLE OF THE TEMPERATURE OF SATURATED STEAM CORRESPONDING TO A GIVEN PRESSURE.

P	*t*°	*P*	*t*°	*P*	*t*°	*P*	*t*°	*P*	*t*°	*P*	*t*°
1	102	21	230	41	269	61	294	81	313	101	329
2	126	22	233	42	270	62	295	82	314	102	329
3	142	23	236	43	272	63	296	83	315	103	330
4	153	24	238	44	273	64	297	84	315	104	331
5	162	25	240	45	275	65	298	85	316	105	331
6	170	26	242	46	276	66	299	86	317	106	332
7	177	27	244	47	277	67	300	87	318	107	333
8	183	28	246	48	278	68	301	88	319	108	333
9	188	29	248	49	280	69	302	89	319	109	334
10	193	30	250	50	281	70	303	90	320	110	335
11	198	31	252	51	282	71	304	91	321	111	335
12	202	32	254	52	284	72	305	92	322	112	336
13	206	33	256	53	285	73	306	93	323	113	337
14	210	34	258	54	286	74	307	94	323	114	337
15	213	35	259	55	287	75	308	95	324	115	338
16	216	36	261	56	288	76	309	96	325	116	339
17	220	37	263	57	289	77	309	97	326	117	339
18	223	38	264	58	291	78	310	98	326	118	340
19	225	39	266	59	292	79	311	99	327	119	341
20	228	40	267	60	293	80	312	100	328	120	341

3. The specific heat of water increases slowly with the temperature, as is shown by the formula (1), page 246. It is usually supposed unity, but at high temperatures the error in doing so is not inconsiderable: thus, at a temperature of 400° F, corresponding to a pressure of 250 lbs. per square inch, the specific heat of water is 1.04; and at a temperature of 300°, corresponding to 67 lbs per square inch, it is 1.02 Where accuracy is required, it therefore cannot be assumed unity at high pressures.

4. The total heat of evaporation of water from 32° at t° is given by the formula.

$$\begin{aligned} \text{Total heat} &= 1091.7 + .305\ (T^\circ - 32^\circ) \\ &\qquad\qquad\qquad\text{(thermal units)} \\ &= 1082 + .305\ T^\circ \\ \text{or}\quad H &= 835{,}300 + 235.46\ T^\circ \ \text{(in foot pounds)} \end{aligned}$$

which formulæ represent with accuracy the results of Regnault's experiments.

The formula commonly used for the latent heat of evaporation, namely,

$$l = 966 - .7\ (T^\circ - 212^\circ) \quad \text{(in thermal units)}$$

assumes that the specific heat of water is unity at all temperatures. If great accuracy is required at high pressures, Rankine's tables should be used, the interpolation in which, for temperatures intermediate to those given in the table, in this case presents no difficulty.

It will be seen hereafter that the quantity of heat necessary to produce a pound of steam from a pound of water depends upon the way in which the steam is produced; it must, therefore, always be carefully remembered that the terms, "total heat of evaporation," "latent heat of evaporation," are conventionally restricted to the case where the steam is formed under *constant pressure.*

5. The external work done in turning 1 lb. of water, at a given temperature, into saturated steam at the same temperature, is $p\ (v-s)$ foot pounds, where v is the volume of steam produced in cubic feet, p the pressure in pounds per square foot, and s the volume of 1 lb. of water or .016 cubic feet. In practice, s is usually neglected, the error so produced being only .8 per cent. at pressure of 226 lbs. per square inch, and much less at lower pressures. When necessary, it is easy to take it into account by subtracting .016 from the given value of v.

The calculation of pv, especially for pressures intermediate between those given in Rankine's tables, is troublesome, and hence a shorter method is desirable. If saturated steam conformed to the perfectly gaseous laws, pv would be given by a formula of the form

$$pv = a\ (461 + t)$$

where a is a constant.

On trial, however, it appears that, except at very low pressures, the errors of this formula are too great to render it of practical use: and if we modify it, writing

$$pv = A + B\ t$$

where A and B are two constants determined by trial, the same is true at temperatures above 212°. Zeuner, however, proposes the formula

$$\text{External work} = p\ (v-s) = A + B\ t - h$$

where A and B are constants determined by trial, and h is the heat (in foot lbs.) required to raise the temperature of 1 lb. of water from 32° to $t°$, a quantity given in Rankine's tables in the column headed h. If the specific heat of water were unity at all temperatures, this formula would be of the same form as the last, h being $772\ (t^\circ - 32^\circ)$: in fact, however, h increases more rapidly than this, and this quicker increase appears on trial very approximately to balance the errors of the original formula.

If we take $A = 15{,}450$ (a value somewhat less than that which corresponds to Zeuner's value) and $B = 846$, the formula becomes

$$\text{External work} = 15{,}450 + 846\ t - h$$

which, as far as the writer has been able to examine it, agrees well with the results derived from Rankine's tables. This formula is very convenient in calculation, as the value of h is readily interpolated for temperatures intermediate to those given in the table. For temperatures below 212° we may write for h, its approximate value $772\ (t - 32)$, and thus obtain

$$\text{External work} = 74\ (542 + t)$$

A still more convenient formula is derived from the formula given in Art. 2 for the density of steam. Supposing the pressure given in pounds per square inch, and the product pv in foot pounds, we have

$$\log\ (pv) = 4\,.675 + .061\,.\ \log\ p$$

The results derived from this formula are sufficiently accurate at all ordinary pressures.

Cycle of Operations. Internal Work.

6. The principle of the "cycle of operations" is usually stated in the following form. Let a substance change its temperature and volume (or more generally its molecular condition) from some given state, and, passing through a complete cycle of such states, return to its original state: then the difference between the whole heat absorbed by the substance in one part of the process, and the whole heat rejected by the substance in another part of the process, is equal to the external work done by the body during the process. For example, let a pound of water be forced into a boiler, there evaporated, then pass through the engine into the condenser, and finally, after rejecting heat into the injection water, become once more water of the same temperature as before: then the difference between the heat absorbed in the boiler and rejected in the condenser by that pound of water, is equal to the effective work done by the engine per pound of steam.

This is a primary axiom of the mechanical theory of heat, evidently true on the supposition of the convertibility of heat and work, because no other effect has been produced, except the external work, the substance being in the same state as before by hypothesis. Its application, however, is easier when it is put in a different and somewhat more simple form. Let the substance change from a state A to a state B, and let Q_1 be the heat expended, U_1 the external work done during the change: let it now change back from B to A by a different set of changes, and let the heat rejected by the substance be Q_2, and the work done upon it by external bodies be U_2, then, by what has been stated above,

$$Q_1-Q_2=U_1-U_2$$

but Q_2 and U_2 are exactly the heat which would have been expended, and the work which would have been done if the substance had changed from A to B by the *second route reversed;* hence, writing the equation in the form

$$Q_1-U_1=Q_2-U_2$$

we see that the principle may be put in the form—

If, from the whole heat requisite to produce a given change of state, the equivalent of the external work be subtracted, then the remainder is always the same in whatsoever way the change be produced.

This form of the principle is in Rankine's work (see p. 304) deduced as a corollary from a different proposition, but it appears to be a necessary conclusion from the original form of the principle (at least whenever some way exists of passing from the state A to the state B, which is capable of being reversed), and it is, in fact, so assumed explicitly or implicitly by other writers. Indeed, it may be questioned whether this form of the principle has not as much claim to be considered axiomatic as the other.

To take an example: Water at a temperature of 32° requires about 904,000 foot pounds of heat to raise its temperature to 293° F and evaporate it at that temperature, which corresponds to a pressure of 60.4 lbs. per square inch, or about 8,700 lbs. per square foot. Its volume is then very nearly 7 cubic feet, and the external work done very approximately 60,900 foot lbs. Subtracting this from 904,000 foot lbs. we get as a remainder 843,100 foot lbs. But now let us suppose that we produce our steam without doing external work. This may be very easily done; we have only to place our pound of water at 32° in a close vessel, the capacity of which is 7 cubic feet, and apply heat till all the water is evaporated; we shall then have a pound of saturated steam at the temperature of 293° F as before, but we shall have done no external work, and the heat expended is therefore no longer 904,000 foot lbs., but 843,100 foot lbs. We have, in fact, in the second case produced the steam and the steam only, while in the first case we have not only produced the steam, but we have also produced 60,900 foot lbs. of mechanical work, and the whole heat expended is separable into the two parts necessary to produce each effect separately.

So it is in the most general case; we must always separate the external and visible work done on external bodies from the internal and invisible work done in changing the state of the body. The second part, which we shall call the internal work, is always the same, however the change is produced.*

In some important cases the body is in motion, and in this case, in applying the principle, any change of kinetic energy must be taken into account. (Comp. Art. 36.)

The authorities on thermo-dynamics do not place the principle considered in this article in the same rank as the principle of the convertibility of heat and work, which is the first principle of the mechanical theory of heat, and the "second principle" which we shall have to consider hereafter. The reason of this apparently is, that it partakes rather of the nature of an axiom than of an experimental law. It is nevertheless admitted by all writers, and the conception by Carnot of a cycle of operations was the first step to a true theory of heat engines.

7. If we separate, in the manner just explained, the latent heat of evaporation of water into two parts, the internal work and the external work, and if we assume for the external work the formula given in 5, namely,

$$\text{External work} = 15{,}450 + 846\ t - h$$

we shall obtain a very simple formula for the internal work, for evidently

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* The expression "internal work" has been hitherto more often used in a different sense: the remainder spoken of in the enunciation is separated into two parts, one of which is described as the heat necessary to produce change of temperature, and the other as the heat necessary to produce change of molecular position, for which second part the term "internal work" has been appropriated. (See, for instance, Tait's "Thermo-dynamics," Art. 46.) But this distinction does not appear to us a useful one, at least at the outset of the subject: the student is thereby compelled to form some hypothesis as to the constitution of matter, whereas "internal work," as defined in the text, is evidently capable of exact definition without any hypothesis at all. We have, therefore, followed Zeuner in his use of the word without entering (as Zeuner does) on the above distinction.

$$\begin{aligned}\text{Internal work} &= \text{Latent heat} - \text{External work}\\ &= \text{Latent heat} + h - 15{,}450 - 846\,t\\ &= \text{H} - 15{,}450 - 846\,t\end{aligned}$$

This internal work may also be called the internal latent heat of evaporation, and, if the term latent heat had been invented after the discovery of the mechanical theory of heat, might very probably have been called simply the latent heat. If we denote it by ρ, then replacing H by its value,

$$\begin{aligned}\rho &= 819{,}850 - 611\,t \quad \text{(foot pounds)}\\ &= 1{,}062 - .79\,t \quad \text{(thermal units)}\end{aligned}$$

Pressure Equivalent to the Expenditure of Heat. Engine Working with Dry Saturated Steam.

8. If A be the area of a piston, κ, the space through which it moves, and p the pressure upon it, then $p\,\text{A}\,\kappa$ is the work done. Let now $\text{A}\,\kappa = 1$, that is, let the piston sweep through 1 cubic foot, then the work is simply p, the pressure supposed expressed in lbs. per square foot. Thus the pressure on a piston in lbs. per square foot may also be considered as the work done per cubic foot swept through by the piston. But we can easily find the heat expended by an engine per cubic foot swept through by the piston, and when expressed in foot pounds, it follows that it may be regarded as a "pressure equivalent to the expenditure of heat," and by division by 144 it may be expressed in lbs. per square inch.

This mode of expressing the expenditure of heat in an engine was introduced by Rankine, and is far more convenient than any other: for once knowing the pressure equivalent to the expenditure of heat, we obtain the actual expenditure of heat in exactly the same way as we obtain the power of the engine from the mean effective pressure: and for this reason, the foot pound is a more convenient unit of measurement in the theory of the steam engine than the thermal unit.

In a non-expansive engine, working with dry saturated steam, the pressure equivalent to the expenditure of heat is easily found as follows:—

Let H_1 be the total heat of evaporation of water *from* the temperature of the feed-water *at* the temperature of the boiler, v_1 be the specific volume of the boiler steam, then H_1 is the heat expended for every pound of steam, and consequently $\frac{\text{H}_1}{v_1}$ for every cubic foot of steam admitted into the cylinder; but for every cubic foot of steam admitted, the piston sweeps through one cubic foot, hence it follows that

$$p_h = \frac{\mathrm{H}}{v_1} \quad \text{(lbs. per square foot)}$$

where p_h is the pressure required.

9. If from the heat expended in an engine we subtract the work done, we get the heat rejected, and hence if p_r be the pressure equivalent to the heat rejected,

$$p_r = p_h - (p_1 - p_3) = \frac{\mathrm{H}_1}{v_1} - (p_1 - p_3)$$

in the case of the non-expansive engine, p_3 being supposed the back pressure.

10. Let us now take the case of an expansive engine working with dry saturated steam, and find the pressure (p_h) equivalent to the expenditure of heat. This may be done in two ways, both involving essentially the same principle.

First Method.—Let the pressure be at admission p_1, at the end of the expansion p_2, and let the corresponding specific volumes be v_1, v_2; then at the end of the stroke we have by hypothesis a cylinder full of dry saturated steam of the pressure p_2, which at exhaust rushes into the condenser (or atmosphere), and is there condensed. Let us now imagine a non-expansive engine working *at pressure* p_2, with the same back pressure p_3; at the end of *its* stroke we shall have a cylinder full of dry saturated steam, which at exhaust will rush into the condenser (or atmosphere), and there be condensed. And, *since the circumstances of the condensation are the same,* the heat rejected will be the same in the two cases, that is,

$$p_r = \frac{\mathrm{H}_2}{v_2} - (p_2 - p_3)$$

where H_2 is the total heat of evaporation *from* the temperature of the feed-water *at* the temperature corresponding to p_2, the final pressure.

Let now $p_m - p_3$ be the mean effective pressure, then the work done by the engine for every cubic foot swept through by the piston is $p_m - p_3$ and hence, since

Heat expended = Heat rejected + Work done

$$p_h = p_r + p_m - p_3 = \frac{\mathrm{H}_2}{v_2} + p_m - p_3$$

Second Method. But we can also reason thus:

Each pound of the steam at the end of the stroke is dry saturated steam of pressure p. However that steam is produced, we must always have

$$\text{Heat expended} = \text{Internal work} + \text{External work},$$

of which two terms the Internal work is always the same, while the External work is the energy exerted by the steam on external bodies during its formation. Now if that steam be formed by the ordinary process of evaporation at constant pressure p_2, then the heat expended will be H_2 the total heat of evaporation *from* the temperature of the feed-water *at* the temperature of evaporation, and we have, therefore, for every pound of steam,

$$\text{Internal work} = H_2 - p_2 v_2$$

but the steam is actually formed by a complex series of changes, such that, instead of overcoming the uniform pressure p_2, it presses on the piston with mean pressure p_m, and exerts on that piston for every pound of steam the energy $p_m v_2$.

$$\therefore \text{ Heat expended} = H_2 - p_2 v_2 + p_m v_2 \quad \text{(per lb.)}$$

and the heat per cubic foot swept through by the piston is given by

$$p_h = \frac{H_2}{v_2} + p_m - p_2 \quad \text{(as before)}$$

The reader should notice that the energy exerted by each lb. of steam, namely $p_m v_2$, may be separated into two parts, of which one $(p_m - p_3) v_2$ is employed in overcoming external resistance, and the other $p_3 v_2$ is expended in work done upon the condensing steam, and reappears in the form of heat in the condenser.

11. The engine being supposed working with dry saturated steam, the equation to the expansion curve is $pv^{\frac{17}{16}} = \text{constant}$, from which, if the ratio of expansion be given, the final pressure p_2, and the mean forward pressure p , are readily determined.

It will not, however, be possible that the steam should be dry and saturated during the expansion, unless heat be supplied by means of a steam or hot-air jacket while the expansion is going on. For suppose the boiler to supply dry saturated steam, then the heat required to produce it will be represented by

$$p^1_h = \frac{H_1}{v_2}$$

for H_1, the total heat of evaporation *from* the temperature of the feed-water *at* the temperature of the boiler, is the heat expended in the boiler per lb. of steam, and therefore $\frac{H_1}{v^2}$ is the heat expended in the boiler per cubic foot swept through by the piston, or p^1_h is the pressure equivalent to the heat expended in the boiler. But p_h is greater than p^1_h, for

$$p_h = \frac{H_2}{v_2} + p_m - p_2$$

$$\therefore p_h - p^1_h = \frac{H_2 - H_1}{v_2} + p_m - p_2$$

$$\doteq p_m - p_2 - \frac{235\ (t_1 - t_2)}{v_2}$$

and it will be found on trial that the expression on the right hand side is always positive. Evidently $p_h - p^1_h$ represents an amount of heat which must be supplied to the steam elsewhere than in the boiler, and if this be not done, the steam cannot be dry at the end of the stroke, but condensation must take place.

When a steam or hot-air jacket exists, moist steam absorbs heat with great rapidity from the hot sides of the cylinder, and experience appears to show that the steam is really prevented from condensing in this way. On the other hand, when the steam is once dry it absorbs heat very much more slowly, and there is consequently good foundation for the supposition that, when the cylinder is jacketed, the steam is dry and saturated during the expansion. But when the cylinder is not jacketed and the steam is not super-heated before it enters the cylinder, then condensation necessarily takes place, even though the cylinder be so well clothed that there is no sensible radiation from its external surface.

The heat produced by friction of the cylinder no doubt modifies this statement to a certain extent. The amount of piston friction is variable and uncertain, but we suppose it much over-estimated in normal circumstances, if we take it as 1 lb. per square inch; this 1 lb. per square inch diminishes the pressure equivalent to the expenditure of heat by the same amount, but such diminution is not sufficient sensibly to reduce the condensation.

Nevertheless the reader must bear in mind that this conclusion is based solely on numerical calculations from the formula just given, and if a different fluid than water were used might not be true. For example, in the case of ether, it is believed that super-heating, and not condensation, would take place.

12. The formula just now given for the pressure equivalent to the expenditure of heat

$$p_h = \frac{H_2}{v_2} + p_m - p_2$$

is true whether or no the steam is dry and saturated at other times, provided only that it is so at the end of the stroke. With this restriction the treatment of the steam may be anything we please; it may contain suspended water, or it may be super-heated, the formula will still be true. The value of p_m of course will be different in each case, but when p_m is known from an indicator diagram or otherwise, p_h can always be determined. We shall consider this further hereafter.

13. By use of the formula given in 5 for pv

$$pv = 15{,}450 + 846\,t - h$$

we can put the formula for p_h in a form in which the calculations from it are very simple. For

$$p_h = \frac{H_2}{v_2} + p_m - p_2$$

$$= p_m + p_2 \left(\frac{H_2}{p_2 v_2} - 1 \right)$$

Now let t_4 be the temperature of the feed-water, then

$$H_2 = 772\ (1082 + .305\,t_2) - 772\ (t_4 - 32)$$

Also since t_2 is never a high temperature, we may put very approximately

$$h = 772\ (t_2 - 32)$$

therefore we have

$$\frac{p_2 v_2}{772} = \frac{15{,}450 + 846\,t_2}{772} - (t_2 - 32)$$

$$= 52 + .096\,.\,t_2$$

$$\therefore\ \frac{H_2}{p_2 v_2} = \frac{1{,}114 + .3\,t_2 - t_4}{52 + .096\,t_2}$$

$$\therefore\ p_h = p_m + \frac{1{,}062 + .21 t_2 - t_4}{52 + .096\,t_2} p$$

The formulæ given by Rankine are

$$p_h = p_m + 15\,p_2 \text{ (for condensing engines)}$$
$$p_h = p_m + 14\,p_2 \text{ (for non-condensing engines)}$$

agreeing well with the results of the foregoing more general formula. These formulæ are very simple and convenient in calculation, as p_m and p_s do not require to be expressed in lbs. per square foot, as they must be in the original form of the formula.

It is, however, necessary to know p_s with great accuracy, any error being multiplied by 15 or thereabouts produces an error in p_h some 15 times as great as an equal error in p_m.

THERMO-MECHANICAL PROPERTIES OF THE PERMANENT GASES.

14. If $p\ v\ p^1\ v^1$ be the pressure and specific volume of air at two temperatures $t\ t^1$, measured by Fahrenheit's scale *on the air thermometer*, it is known that very approximately

$$\frac{pv}{p^1v^1}=\frac{461+t}{461+t^1}\ *$$

It is likewise known that this relation is very approximately true of all other permanent gases, that is, gases which have not yet been liquefied by application of cold and pressure : while, on the other hand, it is less approximately true of gases which are capable of liquefaction, such as carbonic acid. Thus, it is easy to conceive an ideal gas for which this relation is mathematically correct, which ideal gas may be considered as a perfect gas, just as from the properties of water we conceive a perfect fluid, although, in fact, no such thing actually exists.

The above equation represents, then, a relation always existing between the pressure volume and temperature of a perfect gas. It may be written in the form

$$pv=\frac{p^1v^1}{461+t^1}\ (461+t)$$

or

$$pv=c\text{T}$$

where c is a constant which for air has the value 53.15, and T is the† temperature measured by Fahrenheit's scale on a perfect gas thermometer, from a zero 461° below the zero of Fahrenheit's scale, or 493° below the freezing point of water. Evidently the value of c will be proportional to the density of the gas at some given pressure and temperature.

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* The numbers given may probably require slight alterations not exceeding $\frac{1}{200}$th part, but the certainty and importance of the alterations are not sufficient to render it necessary to depart from the values as given by Rankine.

† By absolute temperature we mean at present simply temperature measured in this way.

Again, if air be heated at constant pressure through 1° Fahrenheit measured on the air thermometer, it is found that the quantity of heat absorbed by each pound of the air is very approximately .2375 thermal units, or 183.35 foot pounds, whatever be the pressure or temperature of the gas; in other words, its specific heat at constant pressure when measured in foot pounds is always 183.35. So also it is found that the specific heats at constant pressure of the other permanent gases are always the same, and are, moreover, proportional to their densities, that is, proportional to c very approximately. We are thus led to conclude that these laws are characteristic of the perfectly gaseous state.

Lastly, it is known that the same laws hold good with respect to the specific heat at constant volume of air, and that the ratio of the specific heat at constant pressure to the specific heat at constant volume is 1.408, so that the actual value of the specific heat at constant volume is $\frac{183.35}{1.408}=130.2$. It is true that at present the difficulties of experimenting directly on the specific heat at constant volume of gases have not been overcome, and that the experiments on the ratio of specific heats give discordant results; but for our present purpose there seems no impropriety in assuming the known result, as it will be seen presently we have two independent means of testing the truth of the supposition.

We now observe that the difference of the two specific heats of air is 53.15, that is, it is equal to c; further, as just stated, the specific heat at constant pressure of all permanent gases is proportional to c. We are hence led to the conclusion that, if K_p K_v be the specific heats (expressed in foot pounds) of a perfect gas at constant pressure and constant volume respectively, we must always have

$$K_p-K_v=c$$

as a mathematically exact relation satisfied very approximately by the actual permanent gases.

15. We can now obtain a very important result, namely, that the heat expended in internal work, when any change of state is produced in a perfect gas, is simply $K_v\ (t_1-t_2)$ where t_2 t_1 are the temperatures at the beginning and end of the change.

For let p_2 v_2 p_1 v_1 be the corresponding pressures and volumes of the gas, and let us by application of heat change the volume of our gas from v_2 to v_1, the pressure remaining constantly p_2, then by definition

$$\text{Heat expended} = K_p\ (t - t_2)$$

where t is the new temperature. Also

$$\begin{aligned} \text{External work done} &= p_2\ (v_1 - v_2) \\ &= c\ \ (t - t_2) \\ \therefore \text{Internal work} &= (K_p - c)\ (t - t_2) \\ &= K_v\ (t - t_2) \end{aligned}$$

Next let the pressure be changed from p_2 to p_1 at the constant volume v_1 attained by the first operation; then, by definition,

$$\text{Heat expended} = K_v\ (t_1 - t)$$

and in this case no external work is done, and

$$\therefore \text{Internal work} = K_v\ (t_1 - t)$$

Add together the two values of the internal work, and we get $K_v\ (t_1 - t_2)$ as the internal work done in changing from the original state to the final state.

But this internal work, by the principle of Art. 6, is the same in every case, hence the proposition is true.

Let us now test the truth of this result by the following experiment:—

Air is contained in a vessel which communicates by a pipe furnished with a stop-cock with another vessel, in which a very perfect vacuum exists, or else which contains air of exactly the same temperature but of a different pressure. The stop-cock being opened, air rushes from one vessel to another: after a time the motion ceases, and the temperature is found to be very slightly altered.

This experimental result confirms the truth of what has been stated, for in passing from one vessel to the other no heat is expended on the air, and no external work is done, therefore the change of internal work must be zero: that is, if our proposition is true, the temperature is unaltered.

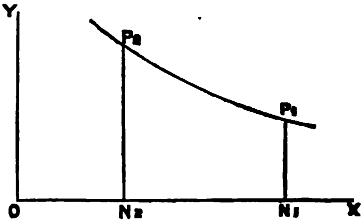
When non-permanent gases are experimented on, the temperature is found to be sensibly changed.

In our mode of statement this experimental result is the first confirmation of the value assumed for the specific heat of air at constant volume. Indeed, it is possible hence to prove, by reasoning analogous to that given above, that the specific heat, at constant volume of air, must have the value assumed by us and no other.

16. We can now find the heat expended in producing any change in a pound of air, for

$$\begin{aligned}\text{Heat expended} &= \text{internal work} + \text{external work}\\ &= K_v\ (t_1 - t_2) + \text{external work}\\ &= K_v\ .\ \frac{p_1v_1 - p_2v_2}{c} + \text{external work}\end{aligned}$$

which is the general equation of the action of heat in a perfect gas. We may represent this geometrically thus: Drawing a diagram, in which as usual ordinates represent pressures and abscissæ represent volumes; let $P_2\ N_2$, $O\ N_2$ be the initial pressure and volume, $P_1\ N_1$, $O\ N_1$ the final pressure and volume. Then the area $P_2\ P_1\ N_1\ N_2$ represents the work done by the expansion of the gas, where $P_2\ P_1$ the expansion curve is perfectly arbitrary, and may be anything we please, if heat be applied according to a proper law. The actual expenditure of heat is geometrically represented by



$$\frac{K_v}{c}\ (\text{rectangle } P_1\, N_1\ .\ ON_1 - \text{rectangle } P_2\ N_2\ .\ ON_2)$$
$$+ \text{area } P_2\ P_1\ N_1\ N_2$$

There are two important cases—

First, let the expansion take place at constant temperature, we then have

$$\text{Heat expended} = \text{Area } P_1\ N_1\ N_2\ P_2\,;$$

but the expansion curve is then a common hyperbola, the asymptotes of which are O X O Y. Then

$$\text{the Area } P_1\ N_1\ N_2\ P_2 = P_1\ N_1\ .\ O\ N_1 \log \frac{O\,N_1}{O\,N_2}$$

the logarithm being hyperbolic,

$$\begin{aligned}\therefore\ \text{Heat expended} &= p_1v_1\ .\ \log r\\ &= c\ T \log r\end{aligned}$$

where r is the ratio of expansion and T is the absolute temperature.

Secondly, let the gas expand in a non-conducting non-radiating cylinder; that is, let it expand without gain or loss of heat, then the expansion curve is a curve called the adiabatic curve, the form of which we require to find. Evidently we have

$$\frac{K}{c}\ (p_2 v_2 - p_1 v_1) = \text{Area } P_1 N_1 N_2 P_2$$

By the aid of the differential calculus we easily find that the curve possessing this geometrical property is given by the equation

$$p\,v^{\gamma} = \text{constant}$$

where $\gamma = \frac{K_p}{K_v}$. This, then, is the equation of the adiabatic curve.

An interesting result has been arrived at by Zeuner, of which this is a particular case: it is easy to show that, if the pressure and volume change in such a way that the specific heat is a constant quantity K, then they must satisfy the equation

$$pv^{n} = \text{constant}$$

where $n = \frac{K - K_p}{K - K_v}$: the adiabatic curve is the special case in which $K = o$.

From the form of the adiabatic curve is obtained a second experimental confirmation of the properties of the specific heat at constant volume assumed in 14: for it can be shown that very approximately the velocity of sound in a gas, when the adiabatic curve is of this form, must be $\sqrt{g\gamma c T}$ (compare Rankine's Art. 252).

When a gas expands without gain or loss of heat, its temperature falls. For suppose it expands from v_1 to v_2, and let its pressure and temperature then be p_1, T_1 and p_2, T_2, then we have

$$p_1 v_1 = c\,T_1\,;\ p_2 v_2 = c\,T_2\,;\ p_1 v_1^{\gamma} = p_2 v_2^{\gamma}$$

$$\therefore\ c\,T_1 v_1^{\gamma - 1} = c\,T_2 v_2^{\gamma - 1}$$

or

$$T_2 = T_1 \cdot \left(\frac{1}{r}\right)^{\gamma - 1}$$

where r is the ratio of expansion, a result which shows that the temperature falls, and enables us to find by how much it has fallen.

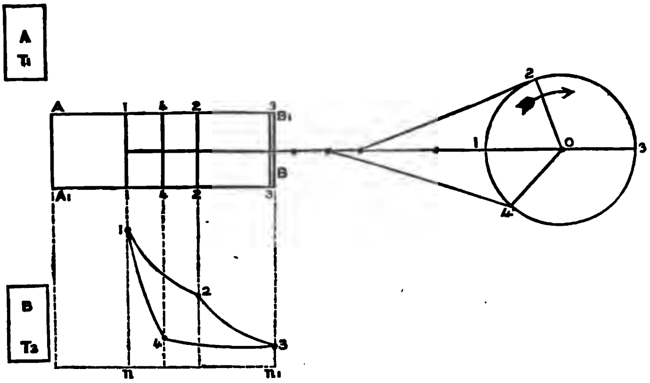
THEORY OF A HEAT ENGINE WORKING WITH A PERFECT GAS. CONDITIONS OF MAXIMUM EFFICIENCY.

17. The simplest kind of heat engine is constructed as follows:—

A B is a working cylinder containing a given quantity, say 1 lb., of a perfect gas always included between the cylinder cover A A^1 and a piston, successive positions of which are represented in the figure by 11, 22, 33, 44, and which may be supposed connected in the usual way with a crank, corresponding positions of which are O_1 O_2 O_3 O_4. The right hand portion of the cylinder between the cover B B^1 and the piston is empty.

A and B are two bodies capable of communicating or abstracting indefinite amounts of heat to any body placed in contact with them and of temperatures (absolute) T_1 and T_2 respectively.

Suppose the crank moving in the direction of the arrow,



and initially let it be in the position O_1, and let the pressure volume and temperature of the gas be then p_1 v_1 T_1 respectively. Then, as the crank moves on, the volume of the gas increases, and if no heat were applied to it the temperature would fall: but this is prevented by placing the body A in contact with the cylinder, which is to be supposed a perfect conductor, so that the slightest depression of the temperature of the gas below T_1 causes heat to flow from A into the gas, and thus the temperature of the gas is maintained constantly at T_1. During this first operation, then, the gas expands at constant temperature, and the expansion curve 1, 2 in the indicator diagram above, is a common hyperbola.

The expansion having reached some convenient point 2, the body A is to be removed from the cylinder, so that no more heat is received by the gas; its temperature then falls, instead of remaining constant, and the expansion curve 2, 3 on the indicator diagram above, is now an adiabatic curve given by $pv^{\gamma}=$constant. This goes on until the piston has reached the end of its stroke and begins to return to compress the air again, and raise its temperature according to the same law by which it fell during the expansion. But this rise of temperature is prevented by the application of the body B, the temperature of which is T_3, the temperature of the gas at the end of the stroke, and which abstracts heat from the gas the instant its temperature rises above T_3.

Thus the gas is compressed at constant temperature T_3, and the compression curve 3, 4 on the indicator diagram given on preceding page, is a common hyperbola.

This compression goes on till the piston reaches a point 4, the position of which will presently be determined when the body B is removed and the temperature of the gas allowed to rise. The gas is then compressed without gain or loss of heat, so that the compression curve 4, 1 on the indicator diagram given on preceding page, is an adiabatic curve. If now the point 4 has been properly taken, the temperature of the gas at the end of the stroke will be T_1, and the gas having returned exactly to its initial state, the process may be repeated as many times as we please.

The required point 4 is easily found thus: let $p_2\ v_2\ p_3\ v_3$ $p_4\ v_4$ be the pressure and volume at the points 2, 3, 4, then since 1, 2, and 3, 4 are common hyperbolas

$$\frac{p_1}{p_2}=\frac{v_2}{v_1} \text{ and } \frac{p_3}{p_4}=\frac{v_4}{v_3}$$

and since 2, 3, and 4, 1 are adiabatic curves

$$\frac{p_2}{p_3}=\left(\frac{v_3}{v_2}\right)^{\gamma} \text{ and } \frac{p}{p_1}=\left(\frac{v_1}{v_4}\right)^{\gamma}$$

Hence multiplying all four equations together

$$v_1\ v_3=v_2\ v_4 \text{ or } \frac{v_2}{v_1}=\frac{v_3}{v_4}$$

or—*the ratio of expansion during the reception of heat must be equal to the ratio of compression during the rejection of heat,* or since we equally have

$$\frac{v_3}{v_2}=\frac{v_4}{v_1}$$

—*the ratio of adiabatic expansion must be equal to the ratio of adiabatic compression.* It is easily seen that each of these last ratios must be equal to $\left(\frac{T_1}{T_3}\right)^{\frac{1}{\gamma-1}}$ which determines their value when the temperatures $T_1\ T_3$ of the bodies A and B are supposed given.

Let us now examine how much work is done by this engine, and at what expenditure of heat.

During the operation 1, 2, the gas is receiving heat from A, and the quantity of heat it receives according to the last article is

$$Q = c\,T_1 \log r$$

where r is the ratio of expansion. During the operation 2,3 which completes the forward stroke, the gas receives no heat, therefore Q is the heat expended in the forward stroke. At the same time the energy exerted on the piston by the gas is represented by the area 1, 2, 3, $n^1 n$.

In the backward stroke the gas rejects the heat into B, and the heat so rejected is

$$R = c\,T_2 \log r$$

At the same time the piston compresses the gas and does work upon it represented by the area 1, 4, 3 $n^1 n$; the engine, in fact, is single-acting and a fly-wheel will be required to carry it through the whole backward stroke.

The difference between these areas, namely, the area of the indicator diagram 1, 2, 3, 4, represents as usual the work done by the engine. We might calculate this area, but it is simpler to apply the principle of the cycle of operations, for since the gas returns exactly to its original pressure, volume, and temperature, we must have

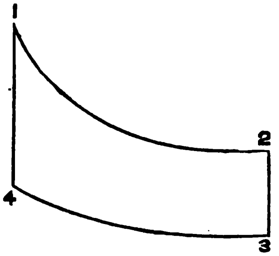
$$\text{Work done} = Q - R = c\,(T_1 - T_2) \log r.$$

The efficiency is now found from the consideration that the heat expended is Q,

$$\text{Efficiency} = \frac{\text{Work done}}{\text{Heat expended}} = \frac{T_1 - T_2}{T_1}$$

18. The simple heat engine just considered may be arranged to work in an indefinite number of other ways, one of which will now be considered as an example.

Let us suppose that the body A remains in contact with the cylinder throughout the forward stroke, and the body B throughout the backward stroke, the indicator diagram will now be as shown in the figure. 1 2 is an hyperbola as before, but now extends through the whole forward stroke, and 3 4 is also an hyperbola as before, but extending through the whole backward stroke; but 1, 4, 2, and 3, are vertical straight lines representing the sudden rise of pressure on contact with the body A, and fall of pressure on contact with the body B, at the commencement of the forward and backward strokes respectively.



To find the efficiency of this arrangement we have, as before

$Q = c\,T_1$ log. r = Heat expended in forward stroke
$R = c\,T_2$ log. r = „ rejected „ backward „

and, as shown in 16, these quantities are also the works done upon the piston by the gas in the forward stroke, and upon the gas by the piston in the backward stroke,

$\therefore$ Work done by engine $= c\,(T_1 - T_3)\log r$, as before

but the whole heat taken away from A, that is, the whole heat expended, is now no longer Q, because at the commencement of the stroke a quantity of heat $K_v\,(T_1 - T_3)$ has to be expended in raising the temperature of the gas from T_1 to T_3, and we consequently have

$$\begin{aligned}\text{Heat expended} &= Q + K_v\,(T_1 - T_3)\\ &= c\,T_1 \log r + K_v\,(T_1 - T_3)\end{aligned}$$

$$\begin{aligned}\text{Efficiency} &= \frac{c\,(T_1 - T_3)\log r}{c\,T_1 \log r + K_v\,(T_1 - T_3)}\\ &= \frac{T_1 - T_3}{T_1} \cdot \frac{1}{1 + \dfrac{K_v}{c \log r} \cdot \dfrac{T_1 - T}{T_1}}\end{aligned}$$

Thus we see that the efficiency of this arrangement is less than that of the other, and we likewise see why it is so, namely, because at certain points in the process the gas has its temperature raised and lowered by contact with the bodies A and B, that is, *that it receives and rejects heat at temperatures sensibly different from the temperatures of A and B.*

There are two conceivable ways of avoiding this loss of efficiency: the first is by storing up the heat rejected during the cooling represented by 2, 3, and employing it to produce the rise of temperature represented by 4, 1. This has been done in actual air engines by the contrivance called the regenerator. (See Rankine, Art. 268.) The second is by inserting an auxiliary heat engine receiving heat from the hot gas of the original heat engine after it has finished its work in that engine; the work of this auxiliary engine will be so much additional work done without any additional expenditure of heat.

19. We shall now show that, by no possible arrangement, with or without the additions just now mentioned, can we obtain an efficiency greater than that obtained by the original arrangement. We might do this by examining

each particular case, but we prefer to reason more generally, as follows:—

Returning to the figure of Art. 17, and supposing initially the crank in the position O_1, let the engine turn in the opposite direction to that indicated by the arrow, neither A nor B being in contact, the gas will expand without gain or loss of heat, and the adiabatic curve 1, 4 will be described on the diagram. As soon as 4 is reached, let the body B be applied to prevent the temperature falling below T_2: heat is continually abstracted from B, the hyperbola 4_1 3 is described on the diagram. Now removing the body A, the piston returns, and the adiabatic curve 3, 2 is described; and, finally, applying A to prevent the temperature rising above T_1, the hyperbola 2, 1 is described, heat all the while passing from the gas into A.

The whole process is now exactly the reverse of what it was; a heat R is abstracted from B, and a heat Q passes into A, while, instead of the engine doing work on external bodies, some force must be applied to the crank which in each revolution will do the work Q—R. And thus, instead of heat passing from A to B, and during its passage a part of it being converted into mechanical energy, we have conversely, by application of mechanical energy, heat passing from B to A, and during its passage mechanical energy converted into heat. In short, the process is *reversible*.

Let us now imagine any other arrangement of this heat engine with or without the additions alluded to just now, and let the heat expended be Q_1, the heat rejected R_1, and let $Q_1-R_1=Q-R$, that is, let the work done by this engine be equal to the work done by an engine of the original arrangement: and let us further suppose that this engine is employed to work the original engine backwards, a thing which we have just shown to be possible.

Now the efficiency of the one engine is $\frac{Q_1-R_1}{Q_1}$ and of the other $\frac{Q-R}{Q}$: hence, if the first be greater than the second, we must have Q_1 less than Q. This, however, is impossible, for the one engine takes away a heat Q_1 from A, and adds a heat R_1 to B, while the other engine takes away a heat R from B, and adds a heat Q to A, the combined action of the two then takes away Q_1-Q from A, and adds an equal quantity R_1-R to B. Thus, if Q_1 be less than Q, heat passes from B to A, and this effect is produced without any expenditure of work, for the combined engines require no power to work them. But this is absurd, for we may take it as an axiom that—*Heat cannot be made to pass from a cold body to a hot one without the expenditure of mechanical energy.**

It will now be asked why we cannot work this combined engine in a reverse direction, and thus show that the efficiency of all possible arrangements must be the same. The answer to this question is, that it is the original arrangement only which admits of being reversed, for in all the other arrangements heat passes from the bodies to the gas, or reversely, at temperatures differing sensibly from that of the bodies themselves, and this is a process which is incapable of being reversed.

For example, suppose the arrangement of Art. 18 worked backwards, instead of the heat $c\ T_2 \log r + K_v\ (T_1 - T_2)$ being taken away from B, and the heat $c\ T_1 \log r + K_v\ (T_1 - T_2)$ being added to A, as would be the case if the process were exactly reversible, it will be found that the heat taken away from B will be $c\ T_2 \log r - K_v\ (T_1 - T\ )$, and the heat added to A will be $c\ T_1 \log r - K_v\ (T_1 - T_2)$; thus, the process not being reversible, the reasoning fails.

Since then no arrangement, even with the addition of a regenerator or auxiliary heat engine, can have a greater efficiency than the original, it follows that without these additions the efficiency must be less, and we finally arrive at the following important conclusions with respect to this kind of engine:—

First.—The maximum efficiency is $\frac{T_1 - T_2}{T_1}$ where T_1 T are the absolute temperatures of the bodies A and B.

Secondly.—The conditions of maximum efficiency of the simple engine are, that the gas shall receive heat at the constant temperature of the body A, and shall reject heat at the constant temperature of the body B. Or, to express the same thing differently, that the process should be *reversible.*

The reader who has followed us so far will now, without much difficulty, be able to proceed further, and to perceive that our reasoning is of much wider application; while, on the other hand, some help is obtained in comprehending what has gone before by the more general considerations to which we now proceed.

SECOND PRINCIPLE OF THE MECHANICAL THEORY OF HEAT. ABSOLUTE TEMPERATURE.

20. To produce mechanical power by means of heat it is

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* Some writers prefer a different form of this axiom. (See Tait's "Thermo-Dynamics," Art. 53.)

in the first place necessary that we should have bodies of different temperatures, for it is evident that if all bodies were of the same temperature, we could no more produce any change by means of heat than we could do work by the action of gravity if all bodies were on the same level. Mechanical energy, then, is produced from heat by the transfer of heat from a hot body to a cold one, and the power of turning heat into work must depend in some manner on the difference of temperature between the hot body and the cold one.

This alone, however, is not sufficient, for if heat simply pass from a body A to a body B, without the intervention of a third body, not only is no mechanical work done, but all the power of turning heat into work due to the difference of temperature of A and B is permanently lost, because the heat is now in the body B and cannot be made to pass back to A.

If, however, we take a third body capable of changing its volume by application of heat, and cause it to receive heat from the hot body and reject heat into the cold body, so that heat is transferred from the hot body to the cold one by its agency, we are then able by means of the changes of volume of this intermediate body to convert a portion of the heat into mechanical energy. It is, however, easy to see that only a portion can be so converted, for it will be found on reflection that rejection of heat into a cold body is just as indispensable a part of the process as reception of heat from the hot body, and thus a part of the heat *necessarily* merely passès through the intermediate body without being converted into mechanical energy. Thus, in the steam engine, the hot body is the hot gas in the furnace, the cold body is the atmosphere or condenser, the intermediate body is water, and the condensation of the steam after it has done its work is just as necessary a condition that work should be done as the evaporation of the water in the boiler.

Thus, for the production of mechanical energy from heat, we must have a hot body, a cold body, and an intermediate body, by the agency of which heat is transferred from the hot body to the cold one.

And since the power of turning heat into work is wasted whenever heat passes from a hot body to a cold one without the agency of an intermediate body, it follows that the process of transformation will be most efficient when the intermediate body has the same temperature as the hot body when receiving heat, and the same temperature as the cold body when rejecting heat. Let us, for instance, suppose that the intermediate body rejects heat at a temperature above that of the cold body, then evidently we can insert an auxiliary heat engine between the intermediate body and the cold body, by means of which a part of that rejected heat can be turned into mechanical energy, without abstracting any additional heat from the hot body. And again, if the intermediate body receives heat from the hot body at a temperature less than that of the hot body, it may evidently be supposed to do so through the agency of an auxiliary heat engine, which receives heat from the hot body and rejects it into the intermediate body, the effect of which is to increase both the work done and the heat expended by *equal* quantities, and thus to increase the efficiency of the process.

Thus we see that the conditions of maximum efficiency of all heat engines are the same as those of the simple kind of engine considered in previous articles.

And more than that, the value of the maximum efficiency must be the same, for it will be seen, on referring back to the last article, that the argument holds equally good if the engine working forwards have the same or a different intermediate body from the engine which is working backwards.

Thus the conclusions stated with reference to the simple engine hold good with reference to any possible heat engine, and these general conclusions constitute the "second principle of the mechanical theory of heat," which may be otherwise briefly stated thus:—

The efficiency of every reversible heat engine is $\frac{T_1-T_2}{T_1}$ *and is greater than the efficiency of any other heat engine working between the same limits of temperature.*

21. Let us now imagine the temperature T_1 of the hot body, or source of heat, to be divided into n equal parts, and let us imagine a quantity of heat Q to flow from that body to a second body, the temperature of which is $T_1\left(1-\frac{1}{n}\right)$, then our results show, that a quantity $\frac{Q}{n}$ of mechanical work is capable of being produced, and that consequently, if such conversion be effected, the quantity of heat $\frac{n-1}{n}.Q$ will pass into the second body. Now imagine a third body, the temperature of which is $T_1\left(1-\frac{2}{n}\right)$

and let this heat pass from the second body to the third body; then the heat capable of being turned into work is—

$$\frac{n-1}{n} \cdot Q \cdot \frac{T_1}{n} \cdot \frac{1}{T_1\left(1-\frac{1}{n}\right)}$$

that is $\frac{Q}{n}$ as before. This process may be continued indefinitely, and we thus see that—*If the temperature of a source of heat be divided into any number of equal parts, then the effect of each of these parts in causing work to be performed is the same.*

It is in this form that Rankine enunciates the "second principle," and his view may be illustrated by the analogy which exists between the difference of level, which causes work to be performed by an hydraulic machine, and the difference of temperature which causes work to be performed by a heat engine. Each foot of fall in a water-wheel is equally effective in doing work by means of the water-wheel, and just so each degree of temperature passed through during the passage of heat from a hot body to a cold one is equally effective in causing work to be done by a heat engine working by means of this heat.

The temperatures in question are, as our investigation shows, to be measured on the perfect gas thermometer, which is, therefore, entitled to be considered as a definite measure of temperature. Temperature, being by its nature incapable of direct measurement, we can only measure it by considering some physical effect which difference of temperature produces: thus the ordinary mode of measurement is, by observing the expansion which bodies undergo when their temperature is raised. This, however, is inconvenient for scientific purposes, since no two thermometers give exactly the same results; for instance, the mercurial and air thermometers, if graduated to indicate correctly the temperatures of melting ice and of water boiling under a given pressure, will be found to differ at intermediate temperatures. We must, therefore, consider some other physical effect due to difference of temperature, and the only one known to be independent of the particular body operated on is the power, of which we have just been speaking, which difference of temperature possesses of converting heat into work. If equal intervals of temperature be understood to mean equal capability of converting heat into work, we get a scale of temperature which may, with propriety, be called *absolute*, and which, as our investigation shows, coincides with that of the perfect gas thermometer, that is (sensibly) with the air thermometer; and the term "absolute," already applied to temperature measured in this way, is hereby justified.

APPLICATION TO THE STEAM ENGINE. RELATION BETWEEN THE LATENT HEAT OF EVAPORATION OF A FLUID AND THE DENSITY AND PRESSURE OF ITS VAPOUR.

22. Thus we see that if a heat engine be perfect, its efficiency is $\frac{T_1 - T_2}{T_1 + 461}$ where T_1 T_2 are the temperatures Fahrenheit, between which the engine works, or if U be the work done in a given time, Q the heat expended in the same time.

$$Q = \frac{T_1 + 461}{T_1 - T_2} \times U$$

hence the expenditure of heat per H P per hour is given by

$$Q = 1{,}980{,}000 \, \frac{T_1 + 461}{T_1 - T_2} \text{ (foot pounds) (1)}$$

a formula which gives the least amount of heat necessary to produce the given power, so long as we are restricted to work within the given limits of temperature. Let us now consider what those limits of temperature are in the case of the steam engine.

The inferior limit T_2 can in no case be less than the temperature of the atmosphere, and in the case of the condensing steam engine must be taken as 100° F., which is the temperature of the condenser. Du Tremblay, indeed, has virtually lowered this temperature to 60° F., or thereabouts, by the addition of an ether engine, working between the temperature of the exhaust steam and the temperature of condensation of ether; but on account of the practical difficulties attending the use of ether, this plan is not likely to come into ordinary use. In the non-condensing steam engine, T_2 is the temperature corresponding to the atmospheric pressure, that is to say, 212° F.

The superior limit T_1 in a simple steam engine, is the temperature of the boiler, for although the hot gases of the furnace have a vastly higher temperature (say t), yet the power of turning heat into work, due to the difference of temperature $t - T_1$, cannot be realised by the arrangements of an ordinary steam engine. To see this clearly, suppose that a fluid existed which, without excessive pressure, could be evaporated at a temperature of 1000° F., and condensed at a temperature somewhat above that of a steam boiler, then it is obvious that an engine could be worked by the vapour of this liquid, which vapour, after exhaustion, could be used to supply heat to the steam boiler. Such an arrangement might not be capable of being practically worked, on account of the high temperatures required, even if the necessary fluid existed, but the conception explains clearly enough the principal reason why the greater part of the heat expended in an ordinary steam engine cannot possibly be turned into work. Hence the superior limit is, as stated above, the temperature of the boiler, or, if the engine work with super-heated steam, the temperature of the super-heater.

Some numerical results of formula (1) will now be given.

HEAT REQUIRED PER HOUR IN A PERFECT ENGINE.

Atm.	Condensing.	Non-condensing.	T_1.
2	9,400,000	38,000,000	249
4	7,800,000	18,800,000	291
6	7,100,000	14,600,000	318
8	6,600,000	12,400,000	340

In the annexed table the first column gives the boiler pressure in atmospheres, and the last the corresponding temperature Fahrenheit: the two other columns give the least amount of heat from which 1-horse power can be produced for the condensing and non-condensing engines respectively. In the condensing engine T_2 the lower limit has been supposed 100° F.

We can now easily deduce the least amount of steam necessary per horse power per hour, for let N be the number of pounds required then,

$$N = \frac{Q}{H_1} \quad (2)$$

where H_1 is the total heat of evaporation of water *from* the temperature of the feed-water *at* the temperature (T_1) of the boiler. It is here supposed that the heat communicated to the steam is all expended in the boiler, and none by means of a hot air or steam jacket or otherwise, and this must be so if the engine is to be theoretically perfect, as will be explained at length further on.

The annexed table gives the numerical results of this formula in the case of the condensing and non-condensing engines respectively, it being supposed, in the first case, that the feed is taken from the condenser so as to have a temperature of about 100° F, and in the second case that the exhaust steam has been employed to raise the temperature of the feed to about 212° F.

POUNDS OF STEAM PER HOUR.			
Atm.	Condensing.	Non-condensing.	T_1.
2	11.2	50.4	249
4	9.2	24.8	291
6	8.3	18.9	318
8	7.7	15.9	340

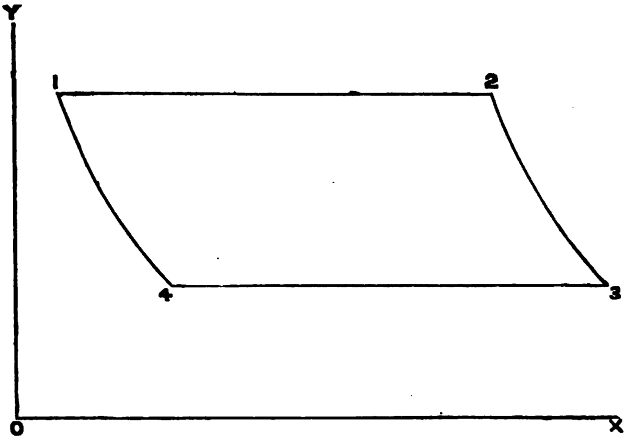
The results of the first table show us that in order to produce 1,980,000 foot pounds of work per hour, we must necessarily expend, under the best circumstances, at least 6,000,000 foot pounds of heat, so that 2-3rds of the heat expended in a steam engine is *necessarily* wasted, the waste arising from the small range of temperature within which the engine works.

The available heat of a pound of average coal burnt in an average steam boiler is about 6,000,000 foot pounds, and the consumption of coal under the best circumstances should be about 1 pound per H P per hour; the smallness of this result, and of the numbers in table 2, which give the quantity of steam theoretically necessary, shows us very distinctly that other causes of loss of efficiency must exist, some of which will be considered hereafter. One main cause of loss of efficiency is, that it is practically impossible to carry expansion to the extent to which it theoretically should be, for maximum efficiency.

23. Since in no case can the quantity of heat capable of being turned into mechanical energy exceed a limit depending on the temperatures between which the engine works, Zeuner has proposed to call the efficiency of a heat engine the fraction which the actual work done by it bears to that available heat, so that, in this way of speaking, the efficiency of a perfect engine is unity. There are some advantages in this nomenclature; but inasmuch as there is a limit to the amount of expansion which can be employed, so that we might as well speak of the available heat in a steam engine working within given limits of temperature with a given grade of expansion, we have not thought it advisable to abandon Rankine's use of the word efficiency. From a passage in the preface to Zeuner's well-known treatise on "Valve Gears," it might be supposed that some divergence of opinion existed between these eminent writers: such, however, is not the case, it is purely a difference of nomenclature.

24. We will now consider the case of a steam engine working under conditions of maximum efficiency.

The figure shows the indicator diagram of an engine of maximum efficiency, O X being the line from which pressures are measured, and O Y the line from which volumes are measured. At the point 1 the pressure and volume of 1 lb. of water in the boiler are represented, which water during admission is evaporated at constant pressure, as represented by the straight line 1 2, the other extremity 2 of which represents the pressure and volume of the steam produced. The steam is then cut off, and expansion takes place without gain or loss of heat till the pressure has fallen to 3, which must be supposed the pressure in a surface condenser. At the end of the stroke the piston returns, and condensation takes place under that same constant pressure.



So far the diagram is exactly the same as an ordinary indicator diagram, in which expansion has been carried to its extreme limit, namely, till the pressure has fallen to the pressure of the condenser. But now, instead of the condensation being complete, we must imagine it stopped at a suitable point 4, and the mixture of steam and water compressed without gain or loss of heat until it becomes once more water of the pressure and temperature of the water in the boiler. Then, if we suppose the condensed steam returned into the boiler, the process may be repeated indefinitely.

Evidently this engine satisfies the conditions of maximum efficiency, for it receives all its heat at the constant temperature (T_1) of the boiler, and rejects all its heat at the constant temperature (T_3) of the condenser. Hence its efficiency must be $\frac{T_1-T_3}{T_1}$, but the heat expended per 1 lb. of steam is L_1, the latent heat of evaporation of water at the absolute temperature T_1, therefore we have

$$\text{Work done per lb. of steam}=\frac{T_1-T}{T_1}\,.\,L_1$$

which work is represented geometrically, as usual, by the area 1, 2, 3, 4 of the indicator diagram.

$$\therefore \text{Area of diagram}=\frac{T_1-T_3}{T_1}\,.\,L_1$$

This equation leads to an extremely important result, for let us suppose that T_1 is very nearly equal to T_3, then the area of the diagram is very nearly

$$(p_1-p_3)\ (v_1-s)$$

where p_1 p_3 are the pressures corresponding to the temperatures T_1 T_3 of saturated steam, v_1 the specific volume of saturated steam at T_1, and s the specific volume of liquid water. Then we have

$$(p_1-p_3)\ (v_1-s)=L_1\,.\,\frac{T_1-T_3}{T_1}$$

Proceeding to the limit, and omitting the suffixes, we see that

$$L=(v-s)\ T\,\frac{dp}{dT}$$

a relation which must be true, not merely for water, but for any fluid whatever. (See Rankine, Art. 255, 256.)

By aid of this equation, the work done in expansion of dry saturated steam can be found exactly, that is, without assuming the approximate formula $pv^{\frac{17}{16}}=$ constant. (See Rankine, Art. 287.)

By aid of this equation, also, the specific volume of steam is calculable when the latent heat of evaporation is known, and the law connecting its pressure with its temperature. (See Rankine, Art. 255, 256.) In the present state of our knowledge it does not seem possible to obtain the density of dry saturated steam in a perfectly satisfactory manner. For it seems to us still an open question, whether the steam experimented on by Regnault, when determining L, was accurately in this condition; while, on the other hand, it is still more difficult to feel satisfied that the steam of Fairbairn and Tate contained no moisture attaching itself to the glass globes in which the experiment was made.

But inasmuch as Regnault's steam was formed under circumstances not unlike those of actual steam engines, we think the calculation volumes cannot be greatly in error for actual steam.

We shall compare the actual steam engine with the perfect steam engine further on, our present object has simply been to prove the above relation.

SUPER-SATURATED STEAM.

25. In an ordinary steam boiler, the steam produced almost always contains more or less water suspended in it, and in this state may be said to be super-saturated. If we suppose that every pound of the steam consists of x pounds of dry saturated steam, and $1-x$ pounds of moisture, then x will be a fraction which, for the sake of a name, we shall call the dryness of the steam, so that when $x=1$, the steam will be perfectly dry, and when $x=o$, it will be wholly water.

Let V be the specific volume of steam, the dryness of which is x, v the specific volume of dry saturated steam, and s the specific volume of liquid water, then evidently,

$$\begin{aligned} V &= (1-x)\, s + x \, . \, v \\ &= x(v-s) + s \end{aligned}$$

or if we write $v-s=u$

$$V = x\, u + s$$

where u is a quantity which may usually be taken equal to v, and, unless x be small, s may be neglected in comparison with ux, so that approximately

$$V = vx$$

Now let us suppose that one pound of water at t_4 is raised to the temperature t_1, and partially evaporated under the corresponding pressure, so that steam is produced, the dryness of which is x_1. Assuming the specific heat of water unity, it is evident that the heat expended is

$$Q = 772\ (t_1 - t_4) + x_1 \, . \, L_1$$

Also the volume of the steam is V_1 and of the water s, therefore the external work done is $p_1\ (V_1 - s) = p_1\ x_1\ u$, and hence it follows that the heat expended in internal work must be

$$\begin{aligned} \text{Internal work} &= 772\ (t_1 - t_4) + x_1 (L_1 - p_1 u_1) \\ &= 772\ (t_1 - t_4) + \rho_1 x_1 \end{aligned}$$

where ρ_1 is the internal latent heat of evaporation, the meaning of which is explained in 7.

And further, if steam of temperature t_1 and dryness x_1 be changed into steam of temperature t_2 and dryness x_2, we shall evidently have for the heat expended in internal work during the change,

$$\text{Internal work} = 772\ (t_2 - t_1) + \rho_2 x_2 - \rho_1 x_1$$

In conformity with the principle of Art. 6, which we have so frequently used, this is to be regarded as the heat necessary to produce the change itself; and to obtain the whole expenditure of heat in producing the change, we have only to add to it the heat expended in doing external work, so that

$$\text{Heat expended} = 772\ (t_2 - t_1) + \rho_2 x_2 - \rho_1 x_1 + \text{External work.}$$

26. We shall now consider a case which does not occur in practice, but is nevertheless of great importance, namely, the expansion of steam in a non-conducting non-radiating cylinder, that is, expansion without gain or loss of heat.

Evidently we shall always have

$$772\ (t_2 - t_1) + \rho_2 x_2 - \rho_1 x_1 + \text{External work} = o$$

or differentiating

$$772\ dt + d\ (\rho x) + p\, d\mathrm{V} = o$$

but $\rho = \mathrm{L} - pu$ and $\mathrm{V} = xu + s$

$$\therefore 772\ dt + d\ (\mathrm{L}x) - d\ (pux) + p\,d\ (ux) = o$$

that is $772\ dt + d\ (\mathrm{L}x) - ux\,dp = o.$

But in 21 we showed that

$$\mathrm{L} = u\mathrm{T}\frac{dp}{dt}$$

where T is the absolute temperature

$$\therefore 772\ dt + d\ (\mathrm{L}x) - \frac{\mathrm{L}x}{\mathrm{T}}\,dt = o$$

Dividing by T and replacing dt by $d\mathrm{T}$.

$$772\frac{d\mathrm{T}}{\mathrm{T}} + \frac{1}{\mathrm{T}}.\ d\ (\mathrm{L}x) - \frac{\mathrm{L}x}{\mathrm{T}^2}d\mathrm{T} = o$$

that is

$$772\frac{d\mathrm{T}}{\mathrm{T}} + d\ \left(\frac{\mathrm{L}x}{\mathrm{T}}\right) = o$$

whence integrating we get

$$772 \log \mathrm{T} + \frac{\mathrm{L}x}{\mathrm{T}} = \text{constant} \quad (1)$$

Suppose the steam to change from the pressure and volume corresponding to T_1, to the pressure and volume corresponding to T, then evidently

$$\frac{L\,x}{T}=\frac{L_1\,x_1}{T_1}+772\ \log\ \frac{T_1}{T}$$

or

$$x=\frac{T}{L}\left(\frac{L_1\,x_1}{T_1}+772\ \log\ \frac{T_1}{T}\right) \quad (2)$$

Since $L=v\,T\frac{d\,p}{d\,T}$ and $V=v\,x$ approximately,* we get for the volume of the steam,

$$V=\frac{1}{\frac{d\,p}{d\,T}}\left(v_1\ x_1\ \frac{d\,p_1}{d\,T_1}+772\ \log.\ \frac{T_1}{T}\right) \quad (3)$$

in accordance with the expression obtained by Rankine in Art. 281 of his work, in which it is to be remarked, that L does not mean the same thing as the L which we have just been using.

The last equation enables us to find the volume of steam which has expanded, until its pressure has fallen from p_1 to p, for T and $\frac{d\,p}{d\,T}$ are both functions of p; the converse problem, however, namely, to find p when the final volume is given, can only be solved by trial, on account of the transcendental form of the equation, and it is therefore desirable to find a formula connecting p and V directly. This was first done by Rankine, who found by numerical calculations from equation (3), that the expansion curve was given approximately by the equation $p\,V^n=$ constant where n is an index to which he ascribed the value $\frac{10}{9}=1\,.\,1\dot{1}\dot{1}$. Zeuner has re-examined the question, apparently with great care and accuracy, and agrees with Rankine in the form of formula, but ascribes to the index n a variable value, namely,

$$n=1\,.\,035+\frac{x_1}{10}$$

depending upon the dryness of the steam in its original condition, with which modification he finds the formula gives results agreeing well with equation (3), so long as the steam does not contain initially more than 30 per cent. of moisture.

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* The exact result is easily written down, but the error of the approximation is of the same order as that introduced by supposing the specific heat of water unity as has been done above. An exact investigation is given by Zeuner, but it is useless to give it here, as special tables are required in computing numerical results.

According to Zeuner, then, the equation to the adiabatic curve is $p\,V^{1.135}$=constant when the steam is originally dry, and $p\,V^{1.105}$=constant when the steam originally contains 30 per cent. of moisture, while, for intermediate states of the steam, the adiabatic curve lies between the two curves given by these equations. If $n=1$, we have the common rectangular hyperbola, and whenever $n>1$, the curve represented by $p\,V^n$=constant falls below the hyperbola. Zeuner's conclusion therefore is, that the adiabatic curve falls below the hyperbola, but approaches it the more nearly the greater the amount of moisture in the steam. We shall return to this presently.

27. We can now find the work done during the expansion of super-saturated steam in a non-conducting non-radiating cylinder, for evidently we have

$$\text{External work}=772\ (t_1-t_2)+\rho_1\,x_1-\rho_2\,x_2$$

where the suffix 1 refers to the steam just before expansion commences, and the suffix 2 to the steam at the end of the expansion. But x_2 is found from equation (2) of the last article, and

$$\rho_1=\mathrm{L}_1-p_1v_1 \quad : \quad \rho_2=\mathrm{L}_2-p_2v_2$$

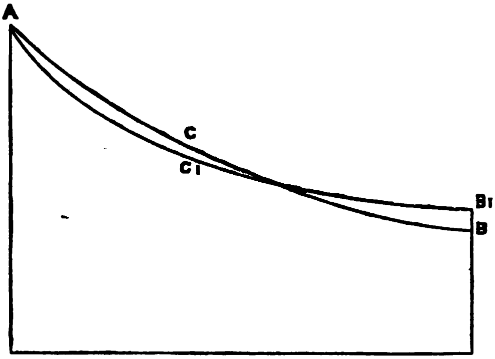
hence, by substitution, we obtain the work done during expansion of the steam, to which we must add the work done during admission of the steam, namely, $p_1\,v_1\,x_1$, in order to get the whole energy exerted on the piston. The same result is obtained by integrating equation (3), and adding $p_2\,v_2\,x_2$. (See Rankine, Art. 284.)

An approximate result is likewise obtainable by finding the mean pressure, on the assumption that the expansion curve is of the form $p\mathrm{V}^n$=constant.

The complete calculations of Rankine (Art. 284), on the supposition that $x_1=1$, will be found to present no difficulty, and can be easily modified to suit the case when x_1 is not unity.

28. In an actual steam cylinder, well clothed so as not sensibly to radiate heat, but not provided with a steam jacket to communicate heat to the steam, the circumstances of the expansion are not the same as those just supposed, for although *on the whole* no heat is gained or lost by the steam, yet evidently at the beginning of the stroke, heat is abstracted from the steam, and at the end of the stroke, heat is given out to the steam, the temperature of the cylinder having some value intermediate between the greatest and least temperatures of the steam. If the law according to which the cylinder exchanges heat with the steam were known, it might be possible to find the expan-

sion curve and the final pressure of the steam. No attempt has yet been made to do this; but we see that the general effect on the curve must be as shown in the figure, where A C B is the adiabatic expansion curve and A C^1 B^1 the actual expansion curve; the pressure in the actual curve will fall more rapidly than in the adiabatic curve at the commencement of the expansion, and less rapidly at the end of the expansion. The question whether the final pressure can ever rise (as shown in the figure) above its value in the adiabatic curve, will be considered presently.



29. The expenditure of heat in an engine working with super-saturated steam, whether provided with a jacket or not, may be found thus:—

The heat expended in internal work, when a pound of water at t_4 is changed into steam of temperature t_2 and dryness x_2, is given by (see Art. 25)

$$\text{Internal work} = 772\ (t_2 - t_4) + \rho_2\, x_2$$

$$\therefore\ \text{Heat expended} = 772\ (t_2 - t_4) + \rho_2\, x_2 + \text{external work},$$

and hence if V_2 be the final volume of the steam in a steam cylinder, the pressure equivalent to the expenditure of heat is given by

$$p_h = \frac{772\ (t_2 - t_4)}{V_2} + \frac{\rho_2\, x_2}{V_2} + p_m$$

but $V_2 = v_2\, x_2$ approximately where v_2 as usual is the specific volume of dry saturated steam, and therefore

$$p_h = \frac{772\ (t_2 - t_4)}{x_2\, v_2} + \frac{L_2}{v_2} + p_m - p_2$$

where L_2 is as usual the latent heat of evaporation of water at temperature t_2.

This formula becomes identical with the one formerly given for dry saturated steam if $x_2 = 1$, and we might reduce it to a simpler form in the same way as was done in Art. 13.

We can now, as in Art. 12, distinguish between the heat supplied in the boiler and the heat supplied otherwise, for let x_1 be the dryness of the boiler steam, t_1 its temperature, then the heat supplied during evaporation per pound of steam must be

$$772\,(t\,-t_4)+x_1\,L_1$$

and the pressure equivalent to this heat is

$$p^1{}_h=\frac{772\,(t_1-t_4)+x_1\,L_1}{V_2=x_2\,v_2}$$

The remainder $p_h-p^1{}_h$ is the heat supplied by a steam or hot-air jacket to the steam while in the cylinder, which heat is given by

$$p_h-p^1{}_h=\frac{L_2-\frac{x_1}{x_2}\,.\,L_1}{v_2}+p_m-p_2-\frac{772\,(t_1-t_2)}{x_2\,v_2}$$

For example, let there be no jacket, then

$$x_2L_2-x_1\,L_1+(p_m-p_2)\,x_2\,v_2=772\,(t_1-t_2)$$

whence,

$$x_2=\frac{772\,(t_1-t_2)+x_1\,L_1}{L_2+(p_m-p_2)v_2} \quad (1)$$

an equation which enables us to find the dryness of the steam at the end of the expansion, if the final pressure of the steam be known, the mean pressure, and the original state of the steam. These quantities, except the original dryness of the steam, can be found from an indicator diagram. Two methods have been used by Hirn to determine x_1, one by using salt water and observing the amount of salt in the condensed steam, another by observing the heat given out when the steam is condensed: in this last method care must be taken to observe with exactness the circumstances of condensation, otherwise the results will be worthless. (Comp. Art. 6.) Supposing this difficulty overcome, comparison of the above formula with experiment would throw a good deal of light on the action of the sides of the cylinder, a question about which much is said, but little is accurately known.

The dryness of the steam at the commencement of the expansion is found from x_2 by considering that

$$x_2\,v_2=r\,x_1{}^1\,v_1 \quad \text{(very approximately)}$$

where r is the ratio of expansion and $x_1{}^1$ is that dryness which of course is less than x, since, during admission, steam is condensed.

30. Till more is known of the action of the sides of the cylinder, it is impossible to say whether it is possible for the

final pressure of the steam to rise above what it would be if the expansion curve were an hyperbola, as it has been sometimes supposed to do by observation of indicator diagrams.

If the sides of the cylinder had no sensible action, and the cylinder were not jacketed, Zeuner's calculations mentioned above show that the curve approaches the hyperbola the more moisture the steam contains, and hence Zeuner concludes that the action of the sides of the cylinder has been over-estimated. This, however, would not account for the cases in which the curve has been supposed to rise up to and above the hyperbola; in a jacketed cylinder this no doubt might happen, if the steam were originally moist; in an unjacketed cylinder the question cannot at present be answered positively.

In actual indicator diagrams the reader must remember that leakage of the valves frequently takes place, and we think it possible that in some cases the effect of clearance may have been overlooked, which effect is very considerable at high rates of expansion.

If the curve be supposed an equilateral hyperbola, the formulæ of Art. 29 enable us to find the dryness of the steam at the end of the admission and at the end of the expansion; but a small deviation from the hyperbolic form will produce a considerable difference in the results.

31. In the foregoing articles we have neglected the cooling of the cylinder by radiation to external bodies, and to the exhaust steam during the return stroke, while on the other hand we have also neglected the effect produced by piston friction and by wire-drawing.

SUPER-HEATED STEAM.

32. The hypothesis that super-heated steam is a perfect gas, is known to be inaccurate even at temperatures of 30° to 50° F above the saturation point. Hïrn has attempted to find a law to which steam should be subject when in the intermediate condition between perfectly gaseous steam and saturated steam. The reader who is interested in this subject should refer to Hïrn's work, or to the chapter on super-heated steam, in Zeuner.

CAUSES OF LOSS OF EFFICIENCY IN A STEAM ENGINE.

33. The principal causes of loss of efficiency in a steam engine may be enumerated as follows:—

(1.) Loss from the narrow limits of temperature within which the engine works.

(2.) Loss from the process imperfectly satisfying the conditions of maximum efficiency.

(3.) Loss from resistance of passages.
(4.) Loss from friction of mechanism.
(5.) Loss by clearance.
(6.) Loss by radiation to external bodies.

The first of these causes of loss of efficiency is fully considered in Article 22, the consideration of the last four is precluded by the plan of the present publication, and there remains for consideration only the second cause of loss, which is the subject of the two succeeding articles.

34. In order that a heat engine may realise the maximum efficiency of which it is capable $\left(\frac{T_1 - T_2}{T_1 + 461}\right)$, the process which the fluid undergoes must satisfy the conditions of maximum efficiency, that is—*The fluid must receive heat at one constant temperature, and reject heat at another constant temperature.* In the steam engine, when working under the most favourable conceivable circumstances, this condition is nearly but not exactly satisfied. For let us suppose that the ratio of expansion is such that the steam, after being cut off, expands until the pressure has fallen from the pressure in the boiler to the pressure in the condenser, and that that pressure corresponds to the temperature of the condenser, which in practice is not exactly the case, on account of the air contained in the steam; let us further suppose that the valves work perfectly, and that there is no loss by resistance of the exhaust passages, so that the back pressure consists solely of the pressure corresponding to the temperature of the condenser: then the steam rejects heat at one constant temperature, and thus one condition is exactly satisfied. The other condition is only approximately so, for although satisfied while the water is being evaporated in the boiler, it is not satisfied while the water is being admitted into the boiler, and its temperature raised from the temperature of the feed to the temperature of the boiler: it is the loss occasioned in this way that we shall here investigate.

Supposing as usual

T_1 = absolute temperature at admission
T_2 = " " " end of expansion
p_1, p_2, the corresponding pressures

x_1 x_2 the weight of pure steam in one pound of the actual steam at beginning and end of expansion.

L_1 the latent heat of evaporation of water at T_1, then we know that the external work done, when one pound of water is evaporated and expanded from pressure p_1 to pressure p_2, is—

$$U^1 = L_1 x_1 \frac{T_1 - T_2}{T_1} + 772\ (T_1 - T_2) - 772\ T_2 \log \frac{T_1}{T}$$

but we know that the greatest amount of work obtainable from a given amount of heat Q is

$$U = \frac{T_1 - T_2}{T_1} Q$$

and if we take Q equal to the total heat of evaporation of water from T_2 at T_1, we get

$$U = \frac{T_1 - T_2}{T_1} \Big(L_1 x_1 + 772 (T_1 - T_2) \Big)$$

It is now evident that $U - U^1$ is the work lost from the process not fulfilling the conditions of maximum efficiency.

$$U - U^1 = 772\ T_2 \,.\, \log \frac{T_1}{T_2} - 772 \frac{T_2}{T_1}\ (T_1 - T_2)$$

Dividing by U we get for the percentage of loss by imperfection of the process

$$\frac{U - U^1}{U} = \frac{772\ T_2 \left(\log \frac{T_1}{T_2} - \frac{T_1 - T_2}{T_1} \right)}{\frac{T_1 - T_2}{T_1} \Big(L_1 x_1 + 772 (T_1 - T_2) \Big)}$$

this formula likewise serves for the high-pressure engine, in which T_2 corresponds with 212° F.

As a numerical example, suppose a high-pressure engine working at a pressure of 5 atmospheres, which corresponds to $T_1 = 767°$ F.; also suppose the steam from the boiler to be exactly saturated, that is, $x_1 = 1$, then, since $T_2 = 212°$ F. $+ 461° = 673°$ F.

$$\frac{U - U^1}{U} = \frac{772 \times 673 \left(\log \frac{767}{673} - \frac{94}{767} \right)}{\frac{94}{767} \left(\times H_1 = 768{,}021 \right)} =$$

but $$\log \frac{767}{673} = .133\ ;\ \frac{94}{767} = .122$$

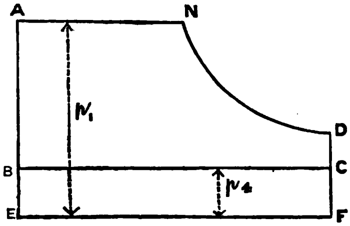
$$\therefore\ \frac{U - U^1}{U} = \frac{772 \times 673 \times 11}{122 \times 768{,}021} = .044$$

In this case, then, the loss by imperfection of the process is about 4½ per cent. of the available heat, a loss which is somewhat increased if the steam be not exactly saturated initially. The effect is, as might be expected, greater in condensing engines, and at pressures of 120 lbs. and upwards rises to 10 per cent. If the feed-water be at a lower temperature than the condensing steam, the effect is of course greater still.

To see how this loss of work might theoretically be remedied, we must consider the difference between the actual series of operations on the steam and the series supposed in a perfect heat engine. In a perfect heat engine the fluid receives heat at constant temperature, expands without gain or loss of heat, rejects heat at constant temperature, and then is compressed without gain or loss of heat till finally it arrives at its initial condition. It is this last part of the process which is omitted in the steam engine; theoretically, instead of forcing water into the boiler, we ought to take the exhaust steam a little before it is all condensed, and compress it without gain or loss of heat until it is all condensed into water of the temperature of the boiler. It is true that work would be spent in producing this compression, and this work would be greater than that lost in working an ordinary feed-pump; but the heat necessary to raise the temperature of the feed-water would be saved, and putting aside friction, the heat so saved would be greater than the work so spent by just the amount lost by the imperfection of the process.

On the whole, however, this cause of loss is an unimportant one compared with others, such as, for instance, that which we next consider, namely, that due to incomplete expansion.

35. The expansion may be said to be complete when it is so great that the final pressure of the steam is that of the condenser (or atmosphere); when this is not the case, the condition of maximum efficiency, that the steam should reject heat at a constant temperature, is violated, and a loss arises. Nevertheless it is seldom or never possible to employ so great an expansion without introducing evils greater than the advantage gained. In practice, a different ratio of expansion r must be employed, and the loss so arising may be thus estimated:—The work done by 1 lb. of steam, admitted at pressure p_1 and expanded r times, is represented by the area A N D F E in the figure in which C F represents p_4, and A E represents p_1; and if the expansion curve be pv^n=constant, that area is given by



$$U''=\frac{p_1 V_1}{n-1}\left(n-\left(\frac{1}{r}\right)^{n-1}\right)$$

where V_1 is the specific volume of the steam originally, which, when the steam is exactly saturated initially, is simply v_1. Subtract from this the area B C F E due to back pressure, which, *for the present purpose*, is supposed simply that corresponding to the condenser temperature, and we get

$$U'''=\frac{p_1 V_1}{n-1}\left(n-\left(\frac{1}{r}\right)^{n-1}\right)-p_4\, r\, V_1$$

which is the actual work done by 1 lb. of steam on the suppositions made.

The maximum amount of work available is given by

$$U=\frac{T_1-T_4}{T_1}\left(L_1\, x_1+772\,(T_1-T_4)\right)=\frac{T_1-T_4}{T_1}\,.\,H_1$$

the suffix 4 now corresponding to the condenser temperature: hence the loss is $U-U'''$ and the percentage of the available heat lost from this cause is given by

$$\frac{U-U'''}{U}=1-\frac{U'''}{\frac{T_1-T}{T_1}H_1}$$

$$=1-\frac{T_1}{T_1-T_4}\cdot\frac{p_1 V_1}{H_1}\cdot\left(\frac{n-(\frac{1}{r})^{n-1}}{n-1}-\frac{p_4\, r}{p_1}\right)$$

This loss includes the loss by imperfection of the process of the last article, and supposes the feed-water to have the temperature (T_4) of the condenser or atmosphere, as the case may be.

As a numerical example, suppose an engine working with boiler pressure $4\frac{1}{2}$ atmospheres, condenser pressure $\frac{1}{10}$ atmosphere, ratio of expansion 6, let the steam be originally saturated, in which case

$$n=1.035+\frac{x_1}{10}=1\,.\,135 \quad \text{(Zeuner's formula)}$$

and we have thence

$$\frac{n-\left(\frac{1}{r}\right)^{n-1}}{n-1}=2\,.\,6\,;\ T_1=760^\circ;\ T_4=115^\circ+461=576$$

$$V_1=v_1=6\,.\,2 \text{ cubic feet};\ H=906000-62000$$

$$p_1=66 \qquad\qquad =844000$$

$$\frac{U-U'''}{U}=1-\frac{76\times 66\times 144\times 62}{184\times 844000}\left(2\,.\,6-\frac{6}{45}\right)$$

$$=1-.\,713=.\,287$$

The two causes of loss explained in this and the last article, may be conveniently classed together, and described as the loss occasioned by the conditions of maximum efficiency being imperfectly satisfied. In the numerical example just given, we see that 29 per cent. of the available heat is lost in this way, and it is, in fact, always a very considerable cause of loss of efficiency.*

FUNDAMENTAL FORMULA FOR THE FLOW OF GASES.

36. The outflow of gases and vapours from a pipe or orifice is a very important question, and will now be briefly considered.

Two elementary methods of proving the fundamental formula are given by Zeuner, both of which are worthy of mention.

First Method.—Conceive two cylinders A and B of different sizes, and two pistons moving in them; B is indefinitely less than A, and moves with a velocity u, while A's motion is very slow. Let the piston pressures be p_1 and p_2.

Every element of the fluid contained between the pistons changes its pressure from p_1 to p_2 in passing from A to B. Suppose one pound to pass, and let its volume while in A be v_1, and when in B v_2, then the work done by the expansion of that pound as it passes from A to B is $\int_{v_1}^{v_2} pdv$.

Also A's piston sweeps through the volume v_1 during the passage of the 1 lb., and the work done on the piston by the pressure p_1 is consequently $p_1 v_1$. Hence the work done by and upon the fluid during the passage of each pound from A to B is

$$p_1 v_1 + \int_{v_1}^{v_2} pdv.$$

Now this amount of mechanical energy is spent, first, in generating visviva in the fluid; secondly, in overcoming the pressure on B's piston.

$$\therefore p_1 v_1 + \int_{v_1}^{v_2} pdv = \frac{u^2}{2g} + p_2 v_2$$

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* The numerical examples of this article and the last are taken from Zeuner's work, from which also the articles themselves are condensed with some modifications.